Micro C - Summer 2015 - Resit Exam Solutions

1. Two firms, 1 and 2, are producing a homogenous good and simultaneously decide on quantity. The price is given by

$$P = 20 - q_1 - q_2,$$

and both firms have a marginal cost of 5.

(a) Find the Nash Equilibrium quantities, under the assumption that both firms are profit-maximizers. What are the profits of the firms in equilibrium?

SOLUTION: Profits for firm i are $\pi_i = (15 - q_i - q_j)q_i$. Taking the first-order condition gives $15 - 2q_i - q_j = 0$. By symmetry, the unique Nash equilibrium will have $q_i = q_j \equiv q^*$, where plugging into the first-order condition yields $15 - 3q^* = 0$, hence $q^* = 5$. Firm-1 profits are therefore $\pi_i = (15 - 2q^*)q^* = 25$.

(b) Continue to assume that Firm 2 wants to maximize profits, but now suppose that Firm 1 is 'irrational' and produces quantity $q_1 = q'$, regardless of what it expects Firm 2 to produce. Suppose furthermore that Firm 2 does not realize that Firm 1 is 'irrational' and instead continues to believe that Firm 1 is a profit-maximizer. What level of irrationality is optimal for Firm 1 (i.e. what value of q' leads to the highest profits for Firm 1)? Describe how Firm 1's quantity and profits at this optimal level compare to those from part (a). Briefly explain the intuition for any similarities or differences (2-3 sentences).

SOLUTION: Since Firm 2 believes that Firm 1 is maximizing profits, Firm 2 will continue to set the Nash equilibrium quantity $q_2 = q^* = 5$. Firm-1 profits are therefore $\pi_1 = (10 - q')q'$. The value of q' that leads to the highest Firm-1 profits is $q' = 5$, which is equal to the Nash equilibrium quantity from part (a), and yields the same profits. The reason for this similarity is that $q_1 = q^*$ is already a best-reply to $q_2 = q^*$, by definition of Nash equilibrium. The fact that Firm 1 is irrational cannot lead to higher profits, since this irrationality (being unobserved) cannot influence Firm 2's strategic decision.

(c) Now assume that Firm 1 is 'irrational' as in part (b), and suppose Firm 2 understands that Firm 1 is irrational. What level of irrationality is optimal for Firm 1 (i.e. what value of q' leads to the highest profits for Firm 1)? Describe how Firm 1's quantity and profits at this optimal level compare to those from parts (a) and (b). Briefly explain the intuition for any similarities or differences (3-4 sentences).

SOLUTION: Firm 2 now realizes that Firm 1 will produce q', so its optimal quantity is given by the best-reply function $q_2 = (15 - q')/2$. To find the value of q' that maximizes Firm-1 profits, plug $q_2 = (15 - q')/2$ into $\pi_1 = (15 - q' - q_2)q'$ to obtain $\pi_1 = (15 - q')q'/2$, which is maximized at $q' = 15/2$, yielding $\pi_1 = 225/8$. These values for Firm-1 output and profits are both higher than those in parts (a) and (b). By committing to produce $q' = 15/2$, Firm 1 is effectively acting as a Stackelberg leader, pushing Firm 2 to reduce its quantity and thereby driving up its own profits.

2. Now consider the infinitely-repeated game $G(\infty)$, with stage game G given by:

		Player 2	
		M	F
Player 1	M	$4, 4$	$-1, 6$
	F	$5, -1$	$0, 0$

Suppose that both Player 1 and Player 2 have discount factor δ. Let (π_1, π_2) denote the average payoff of Player 1 and Player 2 respectively in a particular Subgame Perfect Nash Equilibrium (SPNE). Recall that in particular, player i's average payoff will be π_i in a situation where he obtains a payoff of π_i in every period.

(a) Show for which values of $\delta \in [0, 1)$, if any, a SPNE exists where $(\pi_1, \pi_2) = (4, 4)$.

SOLUTION: Suppose both players use the following trigger strategies: "Play M in period 1. In any period $t \geq 2$, play M as long as the outcome in all previous periods was (M, M), and otherwise play F". Both players earn an average payoff of 4 if they both stick to these strategies. Player 1 has no incentive to deviate from his strategy as long as $4/(1-\delta) \geq 5$, or $\delta \geq 1/5$. Player 2 has no incentive to deviate from his strategy as long as $4/(1-\delta) \geq 6$, or $\delta \geq 1/3$. Hence, these strategies constitute a SPNE if and only if $\delta \in [1/3, 1)$.

(b) Show for which values of $\delta \in [0, 1)$, if any, a SPNE exists where $(\pi_1, \pi_2) = (0, 0)$.

SOLUTION: Suppose both players use the following strategies: "Play F in every period $t \geq 1$, no matter what was played in previous periods". Both players earn an average payoff of zero if they stick to these strategies. No player has an incentive to deviate because (F, F) is a Nash Equilibrium of the stage game. Hence, these strategies constitute a SPNE for all $\delta \in [0, 1)$.

(c) Show for which values of $\delta \in [0, 1)$, if any, a SPNE exists where $\pi_1 = -1$.

SOLUTION: For Player 1 to earn an average payoff of -1, the players must play (M, F) in every period. But Player 1 can guarantee himself a payoff of at least zero by deviating to the strategy "Always play F, no matter what". Hence, there is no value of $\delta \in [0, 1)$ for which a SPNE exists where Player 1 earns an average payoff of -1.

(d) Describe the set of all possible average payoffs (π_1, π_2) that can be obtained in some SPNE, in the limit as δ approaches 1.

SOLUTION: In this limit, the set of all possible average payoffs (π_1, π_2) that can be obtained in a SPNE is given by the set of feasible payoffs from the stage game, i.e. the convex hull of $(4, 4)$, $(5, -1)$, $(-1, 6)$, and $(0, 0)$, subject to the constraints $\pi_1 \geq 0$ and $\pi_2 \geq 0$.

3. Consider a signaling game G' where Nature draws the Sender's type $t \in \{t_1, t_2\}$, the Sender then sends a messages $m \in \{m_1, m_2\}$, and the Receiver responds with an action $a \in \{a_1, a_2\}$. Suppose that from an ex ante perspective, each Sender type is equally likely.

(a) Briefly explain whether G' is a game of complete or of incomplete information (1 sentence)

SOLUTION: By definition, G' is a game of incomplete information, where the Receiver does not observe the Sender's type.

(b) Describe informally the meaning of Signaling Requirements 5 and 6 when applied to G' (3-4 sentences).

SOLUTION: Signaling Requirements 5 and 6 both place restrictions on the out-of-equilibrium beliefs that should be considered 'reasonable' in a Perfect Bayesian Equilibrium. Signaling Requirement 5 says that if the Receiver observes a particular message off the equilibrium path, the Receiver should not believe that the Sender is a type for whom this message is strictly dominated. Signaling Requirement 6 says that following such a deviation, the Receiver should not believe the Sender is a type for whom this message is equilibrium dominated.

(c) **Please answer either part (c) or part (d) in question 3, but not both.** Suppose that a Perfect Bayesian Equilibrium exists in G' where t_1 sends message m_1 and t_2 sends message m_2, both with probability 1. Without any further information, is it possible to say whether this equilibrium satisfies Signaling Requirement 5? Why or why not? (2-3 sentences)

SOLUTION: All messages are played with strictly positive probability in this separating equilibrium. This means that there are no possible messages off the equilibrium path, so out-of-equilibrium beliefs are not relevant. Since Signaling Requirement 5 only places possible restrictions on out-of-equilibrium beliefs, it must be satisfied in such an equilibrium.

(d) **Please answer either part (c) or part (d) in question 3, but not both.** Suppose that a Perfect Bayesian Equilibrium exists in G' where t_1 and t_2 both send message m_1 with probability 1. Suppose furthermore that G' is a cheap-talk game. Without any further information, is it possible to say whether this equilibrium satisfies Signaling Requirement 5? Why or why not? (2-3 sentences)

SOLUTION: In a cheap-talk game, payoffs do not depend directly on the Sender's choice of message. It follows that no Sender type can find a message to be strictly dominated. Hence, any Perfect Bayesian Equilibrium of a cheap-talk game must satisfy Signaling Requirement 5.

(e) Describe a hypothetical real-world situation where Signaling Requirement 6 might give insight into strategic behavior (3-5 sentences).

SOLUTION: One example is education as a costly signal (Spence). There are multiple pooling equilibria where high- and low-ability workers choose the same level of education, but these equilibria do not survive Signaling Requirement 6; loosely put, a worker who deviates to a higher level of education should be able to reveal himself as having high ability. There are also multiple separating equilibria where the low-ability worker chooses the efficient level of education (say Bachelor degree?). Signaling Requirement 6 suggests that the most likely strategic behavior in this situation corresponds to a least-cost-separating equilibrium, where the high type takes just enough education (say Masters' degree?) to leave the low type indifferent about mimicking.

4. Consider a game of incomplete information with Consumer 1 and Consumer 2, where each

consumer's type θ_i is independently drawn from a uniform distribution on $[-1/2, 1/2]$. Consumers must simultaneously choose whether to buy one unit of a good. For each consumer i, buying gives a payoff $u_i = \theta_i + \lambda - p$ if consumer j also buys, and a payoff $u_i = \theta_i - p$ if consumer j does not buy, where $\lambda \geq 0$ and $p \geq 0$ are constants. Not buying always gives a payoff of zero.

(a) Show that a Bayesian Nash equilibrium exists where both consumers buy with probability 1 if and only if $p \leq \lambda - 1/2$.

SOLUTION: Suppose consumer j buys with probability 1. Consumer i will find it optimal to buy himself if buying gives a positive payoff: $\theta_i + \lambda - p \geq 0$. This condition must be satisfied for all $\theta_i \in [-1/2, 1/2]$, which is the case if only if $p \leq \lambda - 1/2$.

(b) Consider a symmetric Bayesian Nash equilibrium where each consumer i buys if and only if his type exceeds a cutoff value: $\theta_i \geq \theta^*$. Show that $\theta^* = (p - \frac{\lambda}{2})/(1 - \lambda)$, and write down the resulting expected total demand from the two consumers.

SOLUTION: Suppose that Consumer j buys if and only if $\theta_j \geq \theta^*$. This means that from the perspective of Consumer i, Consumer j buys with probability $\frac{1}{2} - \theta^*$, since type is uniformly distributed on $[-1/2, 1/2]$. Thus, Consumer i earns an expected payoff of $\theta_i + \lambda(\frac{1}{2} - \theta^*) - p$ from buying. Willingness to pay is strictly increasing in type, so type $\theta_i = \theta^*$ must be indifferent about buying, hence $\theta^* + \lambda(\frac{1}{2} - \theta^*) - p = 0$, or equivalently $\theta^* = (p - \frac{\lambda}{2})/(1 - \lambda)$. Expected total demand is therefore given by $2(\frac{1}{2} - \theta^*)$, or equivalently $(1 - 2p)/(1 - \lambda)$.

(c) Now interpret p as the price set by a seller. What is the value of p that maximizes the seller's expected revenue if $\lambda = 1/2$? What about if $\lambda = 2$?

SOLUTION: First suppose $\lambda = 1/2$. Part (a) then shows that there is an equilibrium where both consumers buy with probability 1 if and only if $p = 0$, but clearly $p = 0$ cannot maximize expected revenues. Hence, for any positive price that yields strictly positive expected revenue, part (b) shows that these revenues are given by $p(1-2p)/(1-\lambda)$, which is maximized at $p = 1/4$. It then follows that $\theta^* = (p - \frac{\lambda}{2})/(1 - \lambda) = 0 \in [-1/2, 1/2]$, so each consumer buys with probability 1/2. Now suppose $\lambda = 2$. Part (b) then shows that expected demand is increasing in p. Thus, the price that maximizes expected revenue can be no lower than the value of p for which expected demand is equal to 2: that is, $(1 - 2p)/(1 - \lambda) = 2$, or equivalently $p = 3/2$. From part (a), $p = 3/2$ is also the highest price for which an equilibrium exists where both consumers buy with probability 1. Hence, $p = 3/2$ maximizes expected revenues when $\lambda = 2$.

(d) What do your answers in parts (a-c) suggest about the relationship between price and expected demand in the presence of network externalities? (3-4 sentences).

SOLUTION: Answers might include the following points. The presence of moderate network externalities (say $\lambda = 1/2$) tends to make demand more sensitive to price, as evidenced by the expression $(1 - \lambda)$ in the denominator of expected demand in part (b). A price increase then reduces consumer willingness to pay, because each consumer expects the other to become less likely to buy. Moreover, when network externalities are strong (say $\lambda = 2$), there can be multiple values of expected demand consistent with a single price. Some of these 'equilibria of the consumer game' can have unusual properties, for example where demand is increasing in price.